



EARLY WARNINGS FOR CATASTROPHIC SHIFTS IN ECOSYSTEMS: COMPARISON BETWEEN SPATIAL AND TEMPORAL INDICATORS

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The task of providing leading indicators of catastrophic regime shifts in ecosystems is fundamental in order to design management protocols for those systems. Here we address the problem of lake eutrophication (that is, nutrient enrichment leading to algal blooms) using a simple spatial lake model. We discuss and compare different spatial and temporal early warning signals announcing the catastrophic transition of an oligotrophic lake to eutrophic conditions. In particular, we consider the spatial variance and its associated patchiness of eutrophic water regions. We found that spatial variance increases as the lake approaches the point of transition to a eutrophic state. We also analyze the spatial and temporal early warnings in terms of the amount of information required by each and their respective forewarning times. From the consideration of different remedial procedures that can be followed after these early signals we conclude that some of these indicators are not early enough to avert the undesired impending shift.

Keywords: Alternate stable states; regime shift; variance; correlation length; patchiness.

1. Introduction

The problem of providing early warning signals of catastrophic shifts in ecosystems has been recently addressed by different methods. Minimal models of lake eutrophication [Carpenter & Brock, 2006; Guttal & Jayaprakash, 2008], lake pollution [Brock & Carpenter, 2006] and lake turbidity [van Nes & Scheffer, 2007] have been used to show that prior

to a regime shift the statistical properties of the time series produced by the models change monotonically. Temporal variance increases [Carpenter & Brock, 2006] and the skewness of the time series probability distribution changes [Guttal & Jayaprakash, 2008]. Similarly, van Nes and Scheffer [2007] showed that the return rates in response to disturbance decrease before the regime shift;

an effect that can be quantified as an increase in autocorrelation of the time series of the observed system variables [Kleinen *et al.*, 2003; Dakos *et al.*, 2008]. In parallel lines, Kéfi *et al.* [2007] studied the transition to desertification in arid ecosystems by analyzing how vegetation patchiness changes with different grazing pressure using a cellular automaton model. The conclusion of their work was that changes in patch-size distributions may be a warning signal for the onset of desertification. Guttal and Jayaprakash [2009] showed that increase variance and a peak in skewness may be regarded as leading indicators of regime shift in spatial ecologies. In all cases, no specific knowledge of the mechanisms underlying the regime shifts is needed to construct the indicators.

Water quality of lakes and reservoirs provide a well-studied example of systems that shift between alternative stable states [Scheffer, 1998; Carpenter *et al.*, 1999; Ludwig *et al.*, 2003; Carpenter, 2003]. We use here a similar example to provide qualitative tools both to anticipate catastrophic changes and to help design management protocols for similar systems. We discuss different proposed spatial and temporal early warning signals of abrupt transitions and we show the link between both kinds of indicators.

In Sec. 2, we describe the model we employ to study early warnings of abrupt changes in the ecological quality of a lake ecosystem. The results obtained with this model are discussed in Sec. 3, where a detailed comparison between spatial and temporal indicators is performed. Finally in Sec. 4, we present our conclusions and suggestions for other ecosystems where these signals could be used to anticipate dramatic changes.

2. The Model

We consider the spatial version of the mean field model of a lake [Carpenter *et al.*, 1999], describing the change over time of some property, s , that characterizes the state of the lake (for example, the water phosphorus concentration). We represent the lake by a square lattice of $L \times L$ sites identified by their coordinates (x, y) ; in our case 100×100 . Obviously lakes of arbitrary shape can be studied by embedding them into a square lattice like the one above, with appropriate boundary conditions. Similarly, we could generalize the problem to the case where several variables determine the status of the lake by considering that s is a vector. In

what follows, for simplicity, we consider the simple schematic square shape and that a single variable s is sufficient to represent the lake status. The evolution equation for the quantity s is then given by:

$$\frac{\partial s(x, y; t)}{\partial t} = a(x, y; t) - bs(x, y; t) + rf[s(x, y; t)] + D\nabla^2 s(x, y; t), \quad (1)$$

where $a(x, y; t)$ represents an environmental factor that promotes s , for instance, the phosphorus loading rate, varying both from point to point and in time, b represents the rate at which s decays, i.e. the nutrient removal rate, r is the rate at which s recovers, i.e. by recycling from the sediment, and f is a Hill function: $f(s) = s^q/(s^q + h^q)$. We have also included a diffusion term as in [van Nes & Scheffer, 2005] with diffusion coefficient D .

We have taken the same parameter value set as in [van Ness & Scheffer, 2005], namely: $b = r = h = 1$, the only difference being that, instead of $q = 4$ we take $q = 8$, as in [Carpenter, 2005]. For the diffusion coefficient, we consider the value $D = 0.1$. We consider that at each time t , nutrient loading rate $a(x, y; t)$ fluctuates around an average value $\bar{a}(t)$ in the interval $[\bar{a}(t) - \Delta, \bar{a}(t) + \Delta]$, where Δ represents the effect of mechanical stirring of the lake (i.e. wind, currents, animals). We have taken $\Delta = 0.125$ and have checked that the results do not depend much on this value. Furthermore, we have assumed that $\bar{a}(t)$ varies in steps of $\delta a = 0.001$ per time step. This value of δa was estimated from [Carpenter, 2005] to represent approximately one year in the evolution of nutrient loading in Lake Mendota, in Wisconsin, USA.

In order to make quantitative comparisons between the different signals, we calculate the following quantities from the time series produced by the model:

- (1) The *spatial variance* of $s(x, y; t)$, σ_s^2 , defined as:

$$\sigma_s^2 = \langle s^2 \rangle - \langle s \rangle^2 = \frac{\sum_{x,y=1}^L s(x, y; t)^2 - \left(\sum_{x,y=1}^L s(x, y; t) \right)^2}{L^2}, \quad (2)$$

where $\langle s \rangle$ stands for the spatial average of the variable s . Note that this quantity requires knowledge of the status of the lake at all points in a grid covering it.

- (2) Similarly to nonspatial models, the *temporal variance* σ_t^2 , at an arbitrary point, say $(x, y) = (0, 0)$, is defined as:

$$\begin{aligned} \sigma_t^2 &= \langle s(0, 0; t)^2 \rangle - \langle s(0, 0, t) \rangle^2 \\ &= \frac{\sum_{t'=t-\tau}^t s(0, 0; t')^2 - \left(\sum_{t'=t-\tau}^t s(0, 0; t') \right)^2}{\tau}, \end{aligned} \quad (3)$$

for temporal bins of size τ as $a(t)$ is varied. In this case, the information required is restricted to only one place in the lake, but taken over a period of time. A typical value of the bin size is $\tau = 20$ years.

- (3) The *patchiness* or *cluster structure*, which helps to understand the behavior of σ_s . Clusters of high (low) s are defined as those connected regions of sites (x, y) such that

$$s(x, y) > s_{\text{crit}}; \quad (s(x, y) < s_{\text{crit}}),$$

where s_{crit} is a predefined critical value of s , which, in our case, lies around $s_{\text{crit}} = 1$.

- (4) *Spatial correlation*, in particular the *two-point correlation function* of the values of s for pairs of cells at (x_1, y_1) and (x_2, y_2) , separated by a distance d , which is given by

$$\begin{aligned} C_2(d) &= \langle s(x_1, y_1)s(x_2, y_2) \rangle \\ &\quad - \langle s(x_1, y_1) \rangle \langle s(x_2, y_2) \rangle \end{aligned} \quad (4)$$

which requires knowledge of the value of the measured quantity at several pairs of points in the lake.

In the following section, we look at the results obtained using this model, in particular, at the evolution of the measured observable $\langle s(t) \rangle$ and at the four quantities defined above.

3. Results

In accordance with previous studies [van Nes & Scheffer, 2005], we have found that, as $a(t)$ goes over a critical value, which lies around $a_{\text{crit}} \approx 0.63$ for our set of parameters, there is a sharp transition in the spatial average of the nutrient concentration, $\langle s(t) \rangle$, as seen in Fig. 1. Once the system has undergone the transition, it is difficult to return to its original, oligotrophic state. Instead, a backward transition is achieved through a hysteresis loop, at a much lower nutrient input rate $a(t)$ (Fig. 1). We

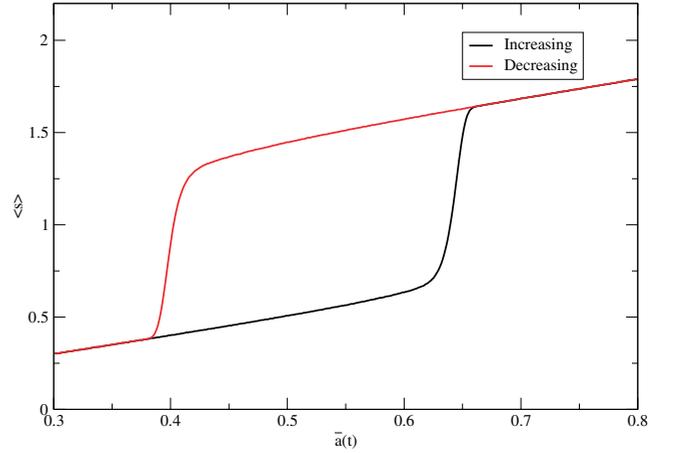


Fig. 1. Average value of the measured observable, $\langle s(t) \rangle$, as a function of the average nutrient input rate, $\bar{a}(t)$. The evolution when the input rate is increasing is shown in black, and, when it is decreasing, in red.

note that the value $s_{\text{crit}} = 1$ approximately divides the hysteresis loop in two equal parts.

In the case of nonspatial models, it has been shown that such transitions are accompanied by an increase in the temporal variance of s , σ_t^2 [Brock & Carpenter, 2006; Carpenter & Brock, 2006]. Given that such increase in variance may be used as an indicator of an oligotrophic lake shifting to eutrophication, one may wonder to what extent similar changes in variance occur in the spatial analog of the model.

Figure 2 is a plot of both the spatial mean nutrient concentration $\langle s(t) \rangle$, and its spatial variance σ_s^2 . It shows that the spatial variance σ_s^2 can also be used as an early warning signal: it starts increasing significantly when the nutrient load $a(t)$ is approximately 0.615 until the point when the transition starts to occur ($a(t) \approx 0.63$). This almost 20-fold increase in spatial variance corresponds to a forewarning of approximately 15 years, since δa is 0.001 per year. After this critical point, spatial variance keeps increasing till it peaks at $a(t) = 0.644$. During this period the lake is in a mixed state, there are cells that are undergoing the transition, before the system as a whole reaches the alternative state at $a(t) \approx 0.66$.

If an increase in spatial variance provides an early warning for transitions in the state of the ecosystem, it is worth checking whether managerial responses based upon such signals may be effective in averting systemic transitions. In Fig. 3 we consider two alternative remedial actions for our lake model. In the first case, when the increase in

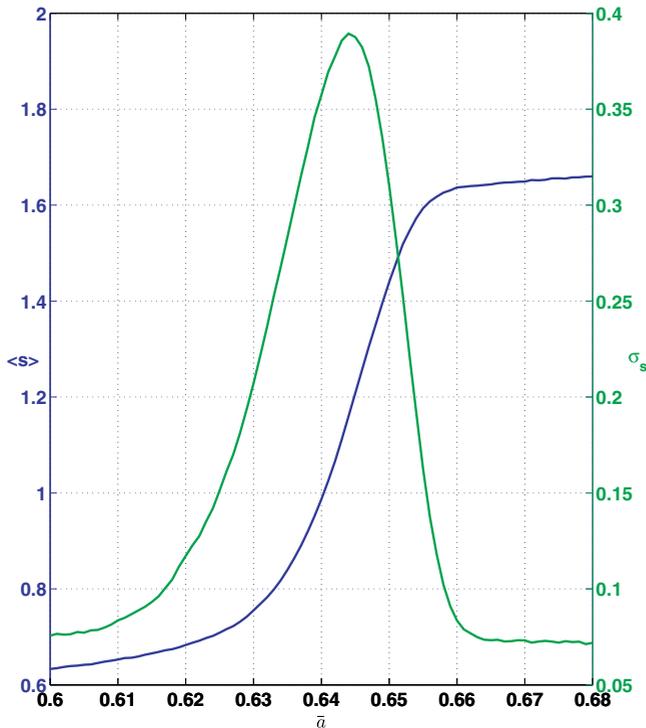


Fig. 2. $\langle s(t) \rangle$ (blue) and σ_s (green), as a function of the average nutrient input rate, $\bar{a}(t)$. We note that the spatial standard deviation provides an earlier warning than the change in the average.

nutrient loading rate $a(t)$ was stopped immediately, the growth in $\langle s(t) \rangle$ nonetheless continued and the lake shifted to the alternative eutrophic state. In the second case, when a much more drastic action is taken, that is reducing nutrient loading $a(t)$ at the same rate as it was previously increasing, the average nutrient concentration $\langle s(t) \rangle$ decreased.

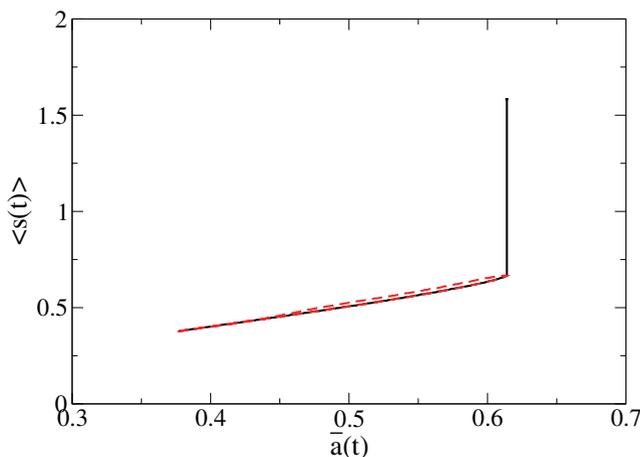


Fig. 3. $\langle s(t) \rangle$ for two remedial actions: decrease the average nutrient input rate, $\bar{a}(t)$ after reaching the load that fires the early warning (red) or keep it constant (black).

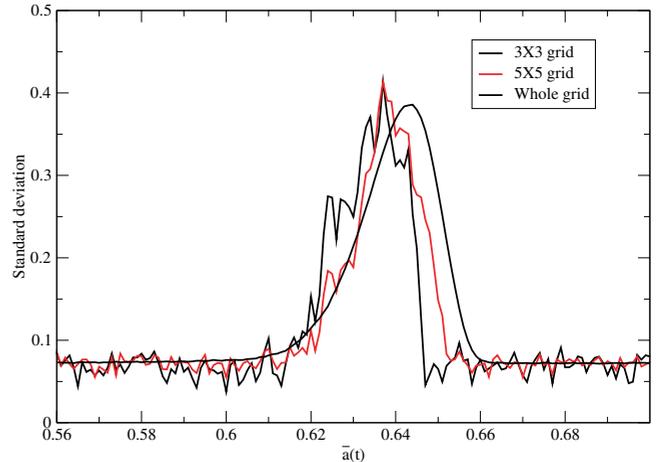


Fig. 4. σ_s computed for different lattice sizes. Even for a small grid of only nine points there is a clear signal.

However, the average nutrient concentration $\langle s(t) \rangle$ was slightly higher compared to the average concentration when nutrient loading was increasing. This means that the lake remained in a mixed state for a long time even under reduced nutrient loading rates. This implies that an even more drastic course of action might be needed to ensure that the lake as a whole remains in its clear oligotrophic state.

We also assessed the practical difficulty of estimating the spatial variance σ_s^2 . In a lattice of 100×100 gridcells, the amount of information needed to estimate the spatial variance is very high. Therefore, we performed calculations over smaller grid sizes (i.e. of 3×3 and 5×5 cells, Fig. 4). We found that the signal does not depend qualitatively on the number of points on the grid that are considered in order to estimate the increase in spatial variance σ_s^2 .

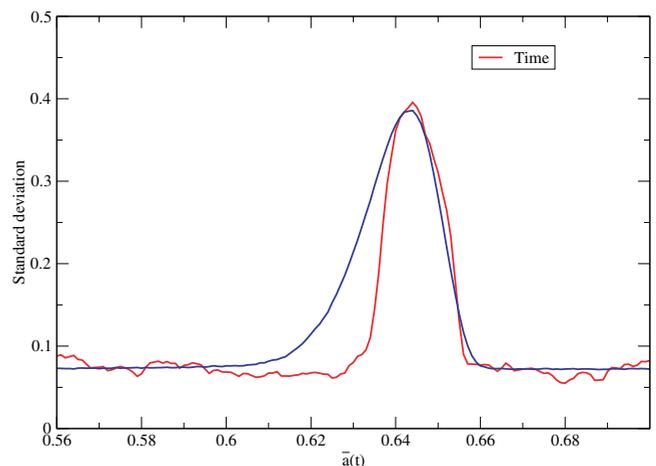


Fig. 5. Spatial (red) and temporal (blue) standard deviations. See text for more details.

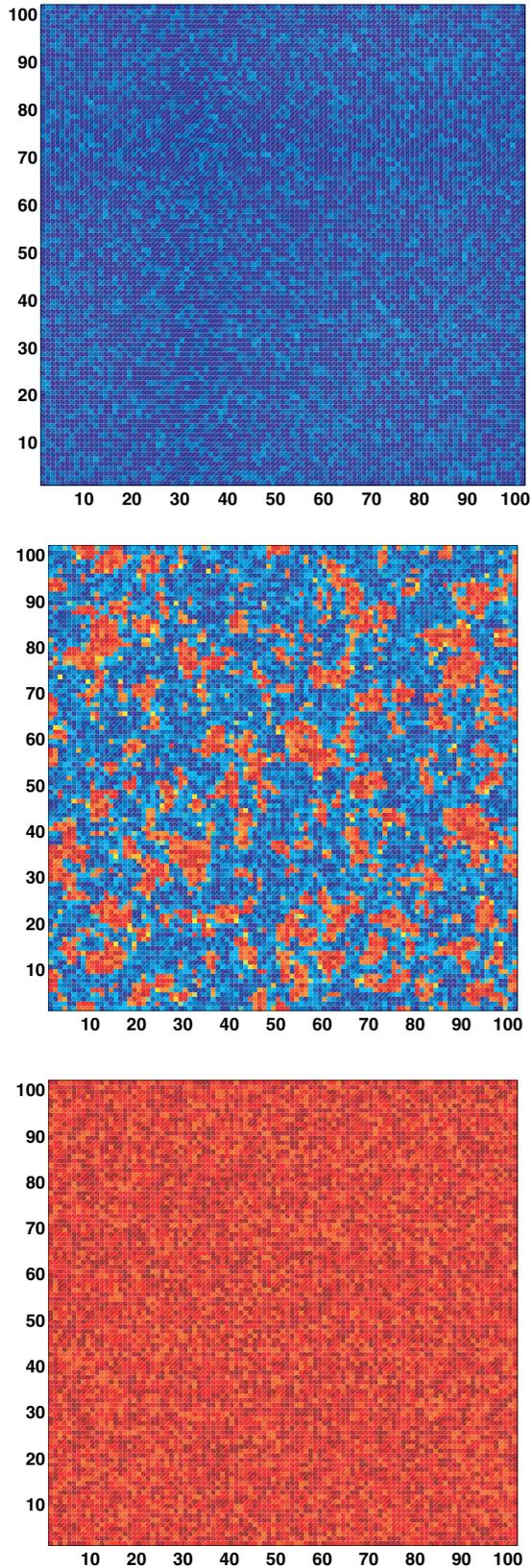


Fig. 6. Values of $s(x, y; t)$ at times $t = 219$ i.e. $\bar{a}(219) = 0.594$ (upper panel), $t = 269$ i.e. $\bar{a}(269) = 0.644$ (middle), $t = 319$ i.e. $\bar{a}(319) = 0.694$ (lower panel), in a color scale ranging from blue ($s = 0.35$) to red ($s = 1.8$).

In Fig. 5, we compare the spatial variance σ_s^2 to the temporal σ_t^2 . To do this, we tracked in time a random individual cell in the lattice and estimated the temporal variance over a sliding window of past values. We found that spatial variance rises earlier than temporal variance and as such it provides a better warning signal for the upcoming transition in $s(t)$. The reason for this is clear: when estimating the temporal variance one must consider past values in the time series, which correspond to situations where the lake is far from undergoing a transition. The spatial variance considers only present values, so if a signal announcing a change is present, it is not obscured by averaging it with data where these indications are absent.

Visualizing the spatial distributions of nutrient concentrations in our lattice offers the basis for understanding why spatial variance rises in our model. As the nutrient load approaches its critical value, nutrient concentrations at individual cells $s(x, y; t)$ start fluctuating spatially (Fig. 6). These increasing fluctuations, which lead to an increase in spatial variance, are caused by two forces. On one hand, each individual cell “slows down” and

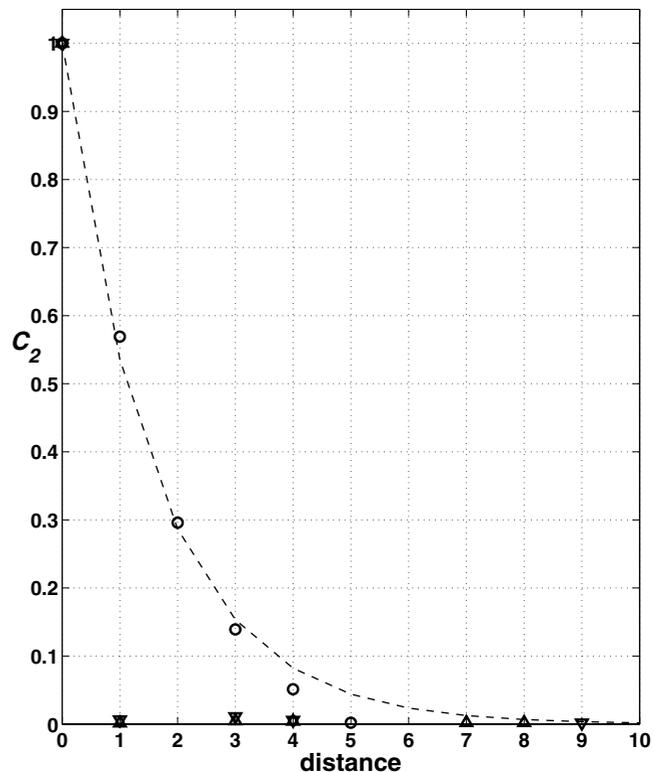


Fig. 7. Two point correlation function $C_2(d)$ for the same number of times as for Fig. 5 and the exponential fit $\exp(-d/\psi)$ with a correlation length $\psi = 1.6$ for $t = 269$ (dashed line) versus the distance d in lattice units.

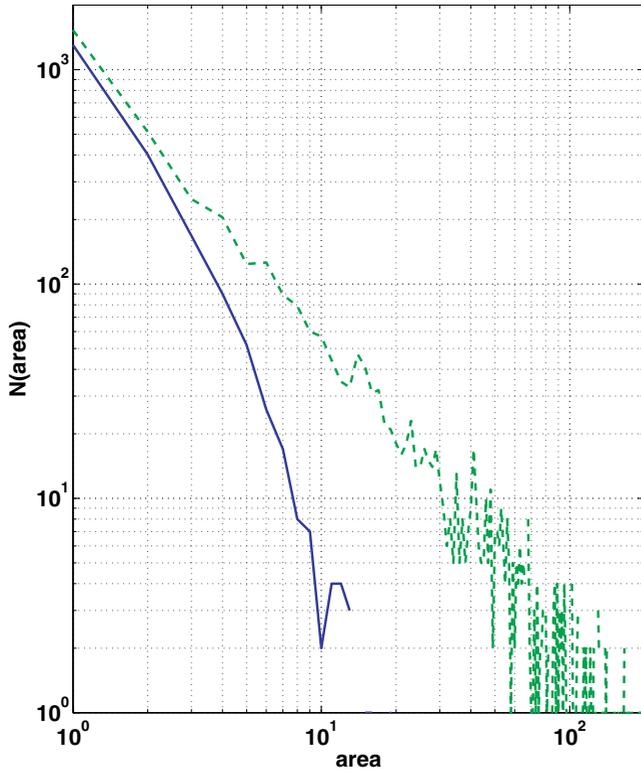


Fig. 8. Cluster size distribution corresponding to $\bar{\alpha} = 0.624$ (blue), and $\bar{\alpha} = 0.644$ (green). This last value corresponds to the maximum of both σ_s and σ_t .

drifts wildly around its deterministic equilibrium as it approaches its bifurcation point [Wissel, 1984; van Nes & Scheffer, 2007]. On the other hand, the heterogeneity introduced by the disturbances on the nutrient loading rate causes cells to achieve a wider distribution of equilibrium values as the critical nutrient load is approached.

Interestingly, as the spatial fluctuations increase, diffusion between cells leads to cluster formation. Clustering creates spatial heterogeneity in the nutrient concentration distribution of the individual cells and can be quantified in terms of spatial correlation. Indeed, when we plotted the two point correlation function $C_2(d)$ (Fig. 7) for the same times considered in Fig. 6, we found that there was no correlation at $\bar{\alpha}(t) = 0.594$, the correlation achieves a maximum value for $\bar{\alpha}(t) \approx 0.644$, at the maximum of the spatial variance, and disappears again for $\bar{\alpha}(t) = 0.694$.

The distribution of clusters in the transition region follows a power law, as illustrated in Fig. 8, for the value $\bar{\alpha} = 0.644$, i.e. at the peak of both σ_s and σ_t . Such a cluster distribution is similar to the usual one found in a second order phase transition at the critical point. It is remarkable that

the one presented here is for a system which is not described in statistical mechanics terms, in particular, where no particular Hamiltonian, partition function or ensemble are given. Before the maximum, for $\bar{\alpha} = 0.624$, when σ_t has not changed and σ_s has increased around one quarter of its way to the maximum, the distribution is not a power law, but resembles an exponential.

4. Conclusions

In this paper we have studied the application of early warning signals for catastrophic transitions in space. For this, we have used the spatial explicit version of a simple lake eutrophication model that shifts locally from an oligotrophic state to a eutrophic state. We have followed the evolution of the spatial variance σ_s^2 by measuring nutrient concentration s on a lattice of points that represented a lake. We found that an increase in spatial variance could serve as an early warning signal of a lake shifting from oligotrophic to eutrophic conditions. Interestingly, sampling even smaller grids, containing fewer points than the whole lattice, proved to be sufficient for observing a similar increase in spatial variance. Additionally, when we compared spatial variance measured at snapshots in time to temporal variance measured locally in one point, we found that spatial variance was a better indicator of the upcoming transition.

The rise in spatial variance is related to the appearance of spatial patterns, in the form of patches of clear (= oligotrophic) and turbid (= eutrophic) water. The identification of such patches of clear and turbid water, for example, by aerial or satellite imaging of a lake surface, and their quantification in terms of spatial 2-point correlation may serve as an alternative way of recognizing a transition from oligotrophic to eutrophic conditions.

However, it seems that the early warning signals considered in our study may not be early enough. The reason is that they do not provide a timely warning of the upcoming change in the lake status which could be avoided by employing simple remedial measures. Instead, it turns out that, in general, when the increase in spatial variance is identified, the lake is already in a partially mixed state where only very drastic actions may be able to avoid a shift to the alternative eutrophic conditions. Such uncertainty will be fundamental for a decision to implement a water quality management program, due to potentially high costs, and shows that there

is still a lot of research needed in order to identify the applicability of early warning signals in ecosystem management [Biggs *et al.*, 2009].

It is worth to remark that the qualitative behavior of our results do not depend strongly on parameter values employed in our model. However the quantitative details of our conclusions depend on the choice of those values. For instance, the value of the diffusion parameter D affects the visual pattern of the patches of clear and turbid water at the transition between the oligotrophic and eutrophic conditions. Furthermore, our model is a simple description of lake water quality. In reality, other variables next to nutrient concentration (i.e. Phosphorus) may also be used to describe the state of a water body, like turbidity, phytoplankton biomass, vegetation cover [Scheffer, 1998]. Nevertheless, our main conclusions should hold even in the more complex case: spatial variance of uni- or multivariate observable quantities could serve as an early warning signal for catastrophic shifts in spatial ecologies outperforming temporal variance.

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